INPUT IMPEDANCE MEASUREMENTS ON WIND INSTRUMENTS USING PULSE REFLECTOMETRY

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ABSTRACT
For many years, frequency domain measurements have been used, with great success, in the analysis of linear, or near-linear, air column musical instruments. In particular, measurements of input impedance have proved extremely valuable in explaining the characteristics of a given instrument. However, it has become increasingly apparent that, for systems with a great degree of non-linearity, measurements made in the time domain may prove significantly more useful.

This paper discusses pulse reflectometry as a time domain technique for finding the input impulse response of an instrument, from which both an instrument bore reconstruction and an input impedance curve can be evaluated.

Impedance curves obtained from pulse reflectometry and from conventional, frequency domain techniques are compared. The advantages and disadvantages of the time domain impedance measurements are discussed, with particular reference to the problem of improving the frequency resolution of the impedance curves. Finally, impedance measurements made on various instruments are presented and discussed.

1. INTRODUCTION
In the acoustical study of musical wind instruments, two types of measurement have proved particularly valuable. One is the experimental evaluation of the input impedance and the other is the direct measurement of the bore profile, using accurate measuring tools. However, both of these measurements can prove problematic. Conventional methods of measuring the input impedance necessitate measuring the volume flow rate (which can be difficult) and bore profile measurements of complicated instruments are impossible when certain sections are inaccessible.

Pulse reflectometry has only recently started to be applied to wind instruments [1, 2], having been developed for seismological studies [3]. It yields the input impulse response of an instrument, from which both the bore profile and the input impedance can be calculated. The advantage of pulse reflectometry is that it requires pressure measurement only (removing the difficulties of volume flow rate measurement) and that it is a non-intrusive technique (removing the problem of inaccessible bores). These factors, and others to be discussed later, indicate the potential of pulse reflectometry as an acoustical measurement technique.

2. BASIC TECHNIQUE
2.1 Determination Of The Input Impulse Response

![Schematic diagram of reflectometer](image)

Figure 1: Schematic diagram of reflectometer

Procedure
An electrical pulse is produced, amplified and used to drive a loudspeaker. The resultant sound pressure pulse (containing frequencies from 0-12kHz) is passed along a copper source tube. A microphone, embedded partway along the tube, records the passage of the input pulse. A short time later, it records the reflections returning from the instrument under test, which is coupled to the far end of the source tube.

Constraints
The source tube length $l_2$ is necessary to ensure that the input pulse has fully passed the microphone before the first of the returning instrument reflections reach it. After the instrument reflections pass the microphone they are further reflected off the loudspeaker, creating source reflections. The source tube length $l_1$ is necessary to separate the instrument reflections from these source reflections. The instrument reflections must be sampled over a time period no
longer than the time taken to travel the distance 2l, to ensure that no source reflections are recorded.

**Deconvolution**

For an ideal delta function sound pressure pulse, the reflections obtained from the instrument would be its input impulse response. However, the sound pressure pulse is not ideal; to obtain the input impulse response, the reflections are deconvolved with the input pulse shape. The input pulse shape is measured by terminating the source tube with a flat plate and recording the reflected pulse (so taking losses along the source tube into consideration) [4].

**2.2 Bore Reconstruction**

Using a suitable algorithm (such as the one derived by Ware and Aki [3]), the bore profile of an instrument can be reconstructed from its input impulse response. Figure 2 shows the bore profile of a stepped cylindrical tube, reconstructed using a layer-peeling algorithm developed by Amir, Rosenhouse and Shimony [5, 6], which compensates for attenuation along the instrument.

**2.3 Input Impedance**

**Theory**

The input impulse response is a measure of the amount of input signal reflected at discrete distances along the instrument bore. As the reflections are caused by changes in impedance within the instrument, it is clear that the input impulse response and the input impedance are closely related. Indeed, the input impedance of an instrument may be evaluated from the input impulse response in the following way [2]:

\[
p(t) = \delta(t) + iir(t)
\]

\[
z_0 \times u(t) = \delta(t) - iir(t)
\]

where \(p(t)\) is the pressure recorded by the microphone at time \(t\), deconvolved with the input pulse shape, \(u(t)\) is the velocity at the microphone at time \(t\), \(z_0\) is the characteristic impedance and \(iir(t)\) is the instrument’s input impulse response.

In the frequency domain this gives

\[
P(\omega) = 1 + II R(\omega)
\]

\[
z_0 \times U(\omega) = 1 - II R(\omega)
\]

where \(P(\omega)\) is the fourier transform of \(p(t)\), \(U(\omega)\) is the fourier transform of \(u(t)\) and \(II R(\omega)\) is the fourier transform of \(iir(t)\).

Hence, the input impedance is given by:

\[
z(\omega)_{in} = \frac{P(\omega)}{U(\omega)} = z_0 \times \frac{1 + II R(\omega)}{1 - II R(\omega)}
\]

**Results**

Figure 3 compares an impedance curve measured using pulse reflectometry with one measured using a conventional frequency domain technique (both curves are for the stepped cylindrical tube displayed in figure 2).

**Discussion**

The two curves are clearly in good agreement in terms of both the amplitudes and frequencies of the peaks and troughs. However, pulse reflectometry has several advantages over conventional frequency domain techniques.

Firstly, pulse reflectometry requires only a pressure measurement whilst frequency domain techniques also require a volume flow rate measurement. This can prove both problematic and time-consuming. Secondly, pulse reflectometry yields
impedance phase information easily. Although phase information can be measured using conventional techniques, the measurement requires two microphones and is not trivial. Finally, the pulse reflectometry system is portable enabling both bore profile and impedance measurements to be made in situ.

One drawback, however, is the limitation on the resolution of the impedance curves. At present, the impedance curve resolution is constrained by the sample period of the instrument reflections which, in turn, is constrained by the source tube length $l_1$. In order to be able to sample over a longer period of time (hence, improving the resolution) either the source tube must be lengthened or the source reflections removed. Increasing the source tube length also increases the attenuation, so it is more desirable to remove the source reflections. This could be done using an active-control type method, allowing impedance curves of very fine resolution to be obtained.

3. IMPEDANCE CURVES OF MUSICAL INSTRUMENTS

To illustrate the use of the techniques in the study of musical instruments, impedance curves for two 19th century cornets are presented. Instead of a mouthpiece, an adaptor of approximately the same volume was used to connect the instrument with the source tube (for the pulse reflectance measurement) or with the measuring microphone and sine wave source capillary.

Figure 4 shows the impedance curves for Rudall Carte ‘Patent Conical Bore’ model cornet in B♭ (EUCHMI 2988), no valves operated. This model is characterised by a narrower and more gently tapering bore between the mouthpiece receiver and the valves (compared with a standard cornet model), the bore diameter increasing through the valve pistons and between one valve and the next. The instrument was designed to play at a pitch of $A_4 = 452.5$ Hz.

Figure 5 shows the impedance curves for Boosey & Co ‘Acme’ model cornet in B♭ (EUCHMI 2704), no valves operated. This instrument has had its playing pitch lowered from $A_4 = 452.5$ Hz to 440 Hz by the insertion of cylindrical extension pieces in its main tuning-slide, and is a standard cornet model with cylindrical bore through the valves.

Comparison of the two curves clearly shows that all the peaks are at slightly lower frequencies for the standard bore cornet which has had its pitch lowered. In fact, analysis of the frequencies of the peaks suggest that the standard cornet should play at a pitch around 116 cents lower than the conical bore instrument. Playing tests confirm that this is indeed the case. It appears that the high pitch instrument was designed to play at $A_4 = 452.5$ Hz with its tuning slides partially withdrawn.

The response of an instrument to a player is largely dependent on the relative heights of the impedance curve peaks (the ‘peak envelope’) and the relative in-tuneness of the frequencies of these peaks. The necessary measurements can be made more easily by pulse reflectance techniques, which also have the advantage that they can be used for bore profile reconstruction measurements and are particularly valuable for historic musical instruments which may be too delicate for extensive playing tests. However, it is clearly evident in figures 4 and 5 that, the frequency resolution of the impedance curves measured using pulse reflectometry is at present a limitation on the accurate analysis of instruments.

References


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